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Experimental observation of noise-induced sensitivity to small signals in a system with on-off intermittency

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Abstract. An experimental (electronic circuit) realization and analytic studies of overdamped Kramers oscillator with an exponential nonlinearity under combined effect of a large multiplicative noise and a small periodic signal were performed. Under certain conditions, when the system exhibits on-off intermittency, it becomes sensitive to very small periodic signals, amplifying them greatly.

PACS. 05.45.-a Nonlinear dynamics and nonlinear dynamical systems – 05.40.-a Fluctuation phenomena, random processes, noise, and Brownian motion – 84.30.-r Electronic circuits

1 Introduction

The role of noise in physical systems in last two decades was subjected a substantial reassessment induced by discovery of numerous interesting phenomena caused by it. Up to now, it is known that in nonlinear systems noise can induce phase transitions [1], complex ordered patterns [2] directed transport of matter [3], as well as facilitate transduction of external signal [4], waves [5] and enhance diffusion [6] in the system.

The next example of constructive role of noise (or, as it is called in [7], noise-induced ordering) is the noise-induced hypersensitivity to small time-dependent signals recently found by us analytically and numerically [8] in a Kramers oscillator with multiplicative white noise. Under effect of large parametric noise the system was able to amplify an ultrasmall (of the order of, *e.g.*, 10^{-20}) deterministic ac signal up to the value of the order of unity. Such an anomalous sensitivity in the system is a result of on-off intermittency [9–17].

On-off intermittency, the phenomenon appearing in a dynamical system when it passes through a bifurcation point under effect of external stochastic time-dependent forcing, attracts now a stable interest of investigators due to its several intriguing properties. The most easily observable of these latter is the specific time behavior of physical quantities: the bursts of large amplitude randomly alternate with the long quiet periods with near-zero amplitude.

On-off intermittency has an extremely important feature of power-law dependence of probability density of burst amplitude [15–17]

$$F(x) \sim x^{\alpha-1}, \tag{1}$$

where α is the scaling index. This expression holds in a wide range of amplitudes $A \ll x \ll 1$, where A is the magnitude of small external signal [8].

For small negative values of α and for vanishingly small external signal ($A \to 0$) we get $F(x) \to \delta(x)$ due to divergence of normalization constant in the $F(x)$.

Now, when

$$A > A_0 = \exp(-1/|\alpha|), \tag{2}$$

it appears that the moments of distribution grow up to the order of unity. Because for $|\alpha| \ll 1$ the value of A_0 is exponentially small, practically any physical value of the signal results in a response of the order of unity. We call this phenomenon hypersensitivity. A similar, but more complex situation appears for small positive α.

In our previous work [8] we found, analytically and numerically, that the simple stochastic system with on-off intermittency, an overdamped Kramers oscillator with multiplicative noise, exhibits such a scaling distribution, and, as a result, demonstrates hypersensitivity to an ultrasmall time-dependent forcing.

One of the main conditions for hypersensitive behavior of the system is a strong nonlinearity of the potential (see below). In the standard Kramers oscillator this nonlinearity is of biquadratic type. However, most of the real physical systems have a nonlinearity not of power-law but (as a rule) of exponential type. Thus, in our circuit we take an exponential nonlinear element and show analytically that an oscillator modified by such a way still exhibits noise-induced hypersensitivity to small signals.

Let us describe now an experimental realization of overdamped oscillator with multiplicative noise. The basic circuit is shown in Figure 1. Its key element is the resistor with negative conductance $G(t)$ which is controlled by the voltage $V_c(t)$. The resistor is designed using an operational

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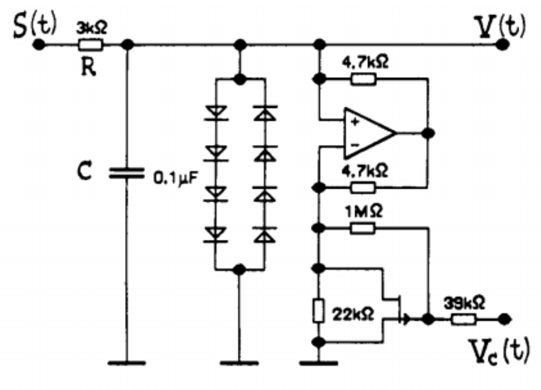

Fig. 1. Electronic circuit described by equation (8). Diodes are of type Si $1N914$, operational amplifier is $\mu A740$, field-effect transistor is $2N5114$.

amplifier and a FET. The voltage V_c that controls the resistor conductance is a sum of the constant bias V_{c0} and the noise $v_n(t)$. This latter is produced by noise generator and has a white spectrum up to frequency about 30 kHz.

The input signal $S(t) = A\,e(t)$, where $e(t)$ is the square-wave zero-mean signal with frequency 0.5 Hz and A is the amplitude, is injected into the system through the resistor R. The static current-voltage characteristics (CVC) of the circuit is shown in Figure 2. We see that for $V < V_0$ the slope of CVC is close to zero. An asymmetry of CVC is caused by technological deviation of parameters of the diodes. The Kirchhoff's law for our circuit is:

$$\frac{S(t) - V(t)}{R} = C\frac{\mathrm{d}V}{\mathrm{d}t} + I_1(V) + I_2(V), \quad (3)$$

where $V(t)$ is the output voltage, $C\mathrm{d}V/\mathrm{d}t$ is the current through the capacitor, and $I_1(V)$ is the current through the nonlinear element (the diode bridge)

$$I_1(V) = I_0(\exp(bV) - \exp(-bV)), \qquad b \sim 1/V_0. \quad (4)$$

The current I_2 through the negative resistance element can be expressed as

$$I_2(V) = G(t)V,$$
$$G(t) = -|G_0| - g(t),$$
$$g(t) = \gamma v_n(t), \quad (5)$$

where $g(t)$ is the conductance noise, with $\gamma \approx 10^{-3}(\Omega\mathrm{V})^{-1}$. Tuning G_0 by the bias value V_{c0}, we set the low conductivity of the circuit (the small slope of the CVC). From equation (3), introducing a dimensionless time, we get:

$$\frac{\mathrm{d}V}{\mathrm{d}\tau} = \lambda V - f(V) + R\gamma v_n(\tau)V + S(\tau), \quad (6)$$
$$\tau = t/RC, \quad \lambda = R(|G_0| - R^{-1} - 2bI_0),$$
$$f(V) = U(\exp(bV) - \exp(-bV) - 2bV), \quad U = RI_0.$$

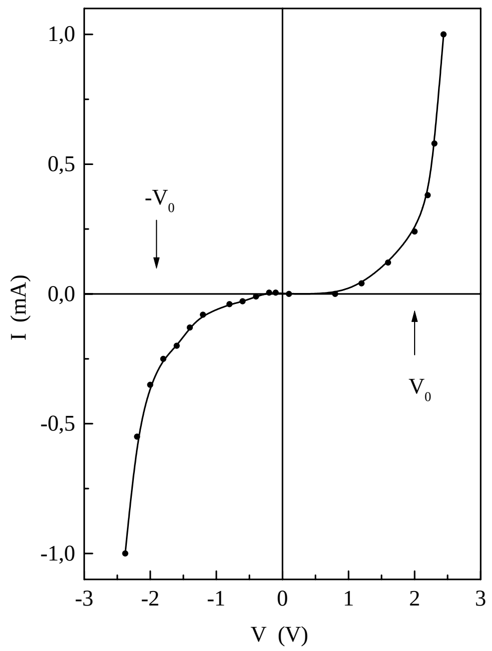

Fig. 2. Static current-voltage characteristics of the circuit. The cutoff voltage is $|V_0| \approx 2$ V. The estimate of α is: $\alpha = -0.22$ for $V > 0$ and $\alpha = -0.49$ for $V < 0$.

The noise correlator is

$$\langle v_n(0)v_n(\tau)\rangle = V_n^2 \exp(-\Gamma RC|\tau|),$$
$$V_n \approx 1.5\,\mathrm{V}, \quad RC = 3\times10^{-4}\,\mathrm{s}, \quad \Gamma \approx 2\times10^5\,\mathrm{s}^{-1}. \quad (7)$$

Earlier [8] we have demonstrated that for white noise approximation the phenomenon of hypersensitivity to small signals takes place for small values of the parameter $\alpha = 2\lambda/\beta^2$, where β^2 is the white noise intensity. Then we can derive from equation (6) $\beta = \sqrt{\frac{2}{\Gamma RC}}R\gamma V_n \approx 0.8$.

From the slope of CVC we can obtain an estimate for λ. Due to asymmetry of the CVC, for positive V this estimate is $\lambda = -0.07$, and for negative V $\lambda = -0.16$. The values of α are -0.22 and -0.49, respectively, and, as a result, the signal gain factors (see Eq. (16) below) differ for the positive and negative parts of the input.

Figure 3 displays an output voltage $V(t)$ for the input signal amplitude $A = 3\,\mathrm{mV}$. We see that the system responds to this small signal by bursts of the order of cutoff voltage $V_0 = 2\mathrm{V}$.

Figure 4 displays the dependence of average gain factor K on the amplitude of input signal A obtained from output power spectrum: $K(A) = \sqrt{S_V(f_0)\Delta f}/A$, where $S_V(f_0)$ is the spectral density for fundamental harmonics of the signal, Δf is the spectral bandwidth. From the slope of this dependence we can obtain $\alpha = -0.2$, close to an estimate obtained from CVC.

Thus, we see that the simple stochastically modulated circuit for small absolute values of parameter α

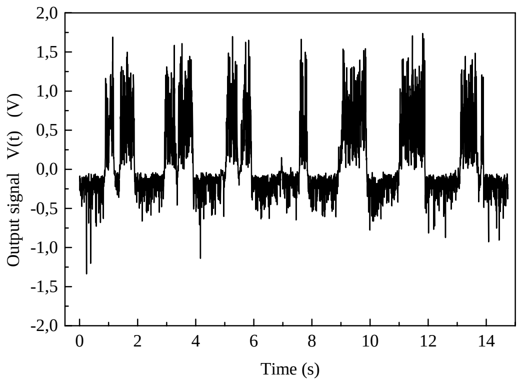

Fig. 3. Output voltage $V(t)$ for input square-wave signal $S(t)$ with amplitude $A = 3$ mV and frequency 0.5 Hz. The asymmetry of $V(t)$ arises from the asymmetry of current-voltage characteristics in Figure 2. As a result, the signal gain factors (see Eq. (16)) differ for the positive and negative parts of the input.

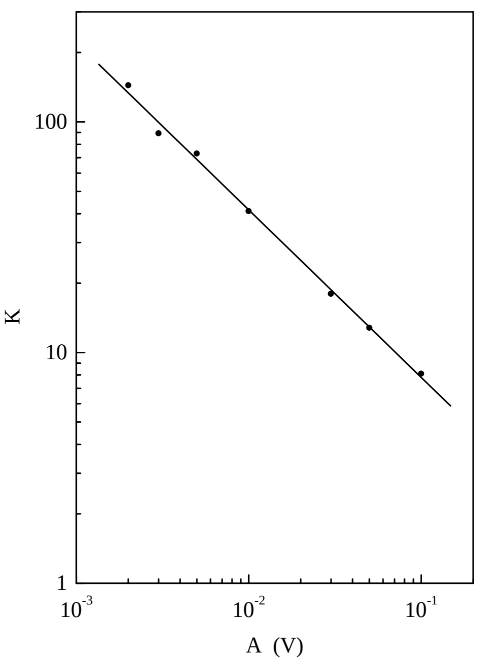

Fig. 4. Average gain factor *vs.* amplitude of input square-wave signal with frequency 0.5 Hz. The solid line is the dependence (16) with $|\alpha| = 0.22$.

demonstrates the noise-induced sensitivity to small time-dependent signals.

We demonstrate also the presence of on-off intermittency in our system.

The common fingerprint of on-off intermittency is a power-law dependence of laminar length

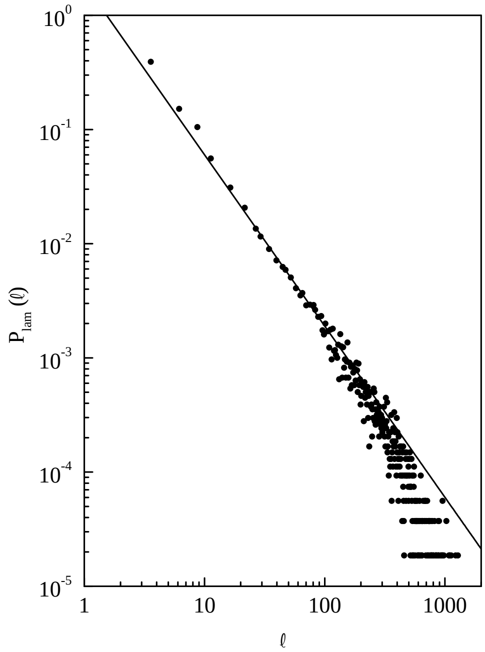

Fig. 5. Laminar length distribution for constant input signal $S(t) = E = -14$ mV and laminarity threshold $p = 0.1$. The solid line is the dependence (8).

distribution [11–14,17]

$$P_{\rm lam}(l) \sim l^{-3/2}, \tag{8}$$

where l is the duration of laminar phase. For $V(t)$ we determine a laminar phase using the condition $V(t) < pV_0$, where p is the laminarity threshold.

Figure 5 presents $P_{\rm lam}$ for constant input signal $S(t) = E = -14\,\mathrm{mV}$ and $p = 0.1$. The experiment shows a good agreement with the dependence (8) (as it is for other values of E and p).

Now we present an analytical treatment of our problem. The equation (6) that describes our circuit differs from that of an overdamped Kramers oscillator in [8]. Therefore, we derive an expression for response of the system (6) to small signal "from scratch".

Physically, the system can be imagined as a Brownian particle in time-dependent potential that chaotically alternate between single-well and double-well form. The stochastic Stratonovich-type equation of nonlinear oscillator with multiplicative noise is:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \lambda x - f(x) + \beta\xi(t)x + Ae(t) + \sigma\phi(t) \tag{9}$$

where $f(x)$ is the nonlinear function, $\xi(t)$ and $\phi(t)$ are the independent Gaussian white noise sources, $e(t)$ is the zero-mean square-wave signal of unit amplitude. The case $f(x) \sim x^3$ was investigated in [1]. In our experiment $f(x)$ is $f(V)$ in equation (6). Solving a Fokker-Planck equation

$$\frac{\partial F}{\partial t} = -\frac{\partial}{\partial x}\left(\left(\lambda + \frac{\beta^2}{2}\right) x - f(x) + Ae(t)\right) F + \frac{1}{2}\frac{\partial^2}{\partial x^2}(\beta^2 x^2 + \sigma^2)F \quad (10)$$

for the case $(A, \sigma) \ll (\lambda, \beta, b, U)$ and using adiabatic approximation (the period of signal is taken much larger than the characteristic time of establishing of stationary probability density function (PDF) in the system) we obtain an expression for PDF:

$$F(x,t) = N(x^2 + \frac{\sigma^2}{\beta^2})^{(\alpha-1)/2} \times \exp\left(\frac{2Ae(t)}{\beta\sigma}\arctan\frac{\beta x}{\sigma} - \frac{2}{\beta^2}\Phi(x)\right),$$

$$\alpha = \frac{2\lambda}{\beta^2}, \quad \Phi(x) = \int_0^x \frac{f(x)}{x^2}\mathrm{d}x = Ub\sum_{k=1}^{\infty}\frac{(bx)^{2k}}{k(2k+1)!}\cdot \quad (11)$$

In the case of negligibly small additive noise ($\sigma = 0$)

$$F(x,t) = N|x|^{\alpha-1}\theta\big(\mathrm{sign}(Ae(t)x)\big) \times \exp\left\{-\frac{2Ae(t)}{\beta^2 x} - \frac{2}{\beta^2}\Phi(x)\right\}, \quad (12)$$

where θ is the Heaviside step function. One can obtain from equation (12) the PDF of scaling type in a wide interval $A \ll |x| \ll 1$:

$$F(x,t) \sim |x|^{\alpha-1}\mathrm{sign}(Ae(t)x). \quad (13)$$

The normalization constant N for our case ($|\alpha| \ll 1$ and $U/\beta^2 \sim 1$) is

$$N = \begin{cases} \alpha & \alpha > 0, \quad z \gg 1, \\ 1/\ln\dfrac{1}{A} & \quad z \ll 1, \\ |\alpha|A^{|\alpha|} & \alpha < 0, \quad z \gg 1, \end{cases} \quad (14)$$

where $z = |\alpha|\ln\frac{1}{A}$.

The power-law dependence of PDF is one of the fingerprints of on-off intermittency [15–17]. The other characteristic feature of equation (12) is its sensitivity to the sign of the signal $e(t)$. From equation (12) we get for mean and mean square values taking for simplicity $\beta,\ U,\ b \sim 1,\ |\alpha| \ll 1$ and, for instance, $z \ll 1$:

$$\langle x(t)\rangle \sim e(t)/\ln(1/A),$$
$$\langle x^2(t)\rangle \sim 1/\ln(1/A). \quad (15)$$

Then the gain factor is :

$$K = \frac{\langle x(t)\rangle}{Ae(t)} \sim \begin{cases} (A\ln\frac{1}{A})^{-1} & z \ll 1, \\ |\alpha|/A^{1-|\alpha|} & z \gg 1,\ \alpha < 0. \end{cases} \quad (16)$$

From equation (16) we see that the gain factor K is very large for small α and A, *i.e.*, the system is hypersensitive to small signal $Ae(t)$.

To conclude, we enumerate the main conditions for hypersensitive behavior of the model: i) large fluctuations of the bifurcation parameter (the coefficient in the linear term in Eq. (9)) ($\beta^2 \gg |\lambda|$, *i.e.*, $|\alpha| \ll 1$); ii) sufficiently strong nonlinearity of the potential; and iii) adiabaticity of input signal, *i.e.*, the period of signal should be much larger than the system relaxation time (the time of establishing of stationary PDF). We managed to realize these conditions in our experiment.

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